Enrollment No……………..

Master of Science (Mathematics)
Fourth Semester Main Examination, June-2021
Functional Analysis-II [MSM401T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Define orthonormal sets. Prove that an orthonormal set is linearly independent.
(b) State and prove uniform boundedness principle.

OR

(a) Define ' Direct sum of two subspace of a vector space X. prove that if y be any closed subspace of a Hilbert space h. then

$$H = y \oplus z \text{ where } z = y^1$$

(b) Show that the norm of an isometry is one.

Q.2 (a) A liner operator S: f $\rightarrow$ f is defined by S $\{x_1 x_2, ----\} = \{o, x_1, x_2, - - -\}$ find its adjoint S^t
(b) Let H be a separable Hilbert space then prove that every orthonormal set in H is countable.

OR

(a) State and prove Riesz representation theorem.
(b) Define total orthonormal. Let u be a subset of an inner product space x, which is total in x. then prove that x ⊥ M⇒ x = 0

Q.3 (a) Let x be a normal space over C let o≠a ∈x show that these is some functional f on x such that.

f (a) = IIaII and IIfII = 1

(b) Show that the unitary opertors on a Hilbert spacer H from a group.

OR

(a) Let T:H ⇒ H be a bonded linear operator on a Hilbert space H. Then prove that if T is self adjoint $< T_x, x >$ is real for all $x \in H$
(b) Explain Hillbert Adjoint operators.

Q.4 (a) If a normed space X is reflexive, show that X' is reflexive.
(b) State and prove uniform boundedness theorem.

OR

(a) Explain strong and weak convergence.
(b) State and prove category theorem.

Q.5 (a) Show that an open mapping need not map closed sets onto closed sets.
(b) Prove that a bounded linear operator. T from a Banach space X onto a Banach space y has the property that the image $T(B_o)$ of the open unit ball Bo= B(0,1) ⊂ X contains an open ball about $0 \in y$.

OR

(a) Define closed linear operators.
(b) State and prove closed graph theorem.

Enrollment No………………

Master of Science (Mathematics)
Fourth Semester Main Examination, June-2021
Advanced Special Function-II [MSM402T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Each question has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Derive the Rodrigue's formula.

$$P_n'(x) = \frac{1}{2^n n!} \frac{a^m}{dx^n} (x^2 - 1)^n$$

(b) Show that

$$(n + 1)L_{n+1}(x) = (2_{n+1} - x)L_n(x) - nL_{n-1}(x)$$

OR

(a) Define Laguerre Polynomial Prove that

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n} (x^n . e^{-x})$$

(b) Prove that: $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$

Q.2 (a) Prove that

$$\int_0^\infty x^{\propto} e^{-x}\, t_n \propto x\, L_m \quad {}^{\propto}(x)dx = 1 + \frac{(n + \propto + 1)}{n!} \sigma n, M$$

(b) Express

$\text{H}n(x) = x^4 + 2x + 2x^2 - x - 3$. In terms of Hermite's polynomial

OR

(a) Prove that

$$D\{L_n{}^{(\propto)}(x)\} = D\{L_{n-1}{}^{(\propto)}(x)\} - L_{n-2}{}^{(\propto)}(x)$$

(b) Expand $x^3 + x^2 - 3x + 2$ in a series of Laguerre polynomial.

Q.3 (a) Show that

(i) $L_n(0) = -n$ $(ii)\ L_n''(0) = \frac{\{n(n-1)\}}{2}$

(b) Give trigonometric definite of Chebyshav polynomials.

OR

(a) Prove that

$$\sum_{n=0}^{\infty} t^n L_n^{(\propto)}(x) = \frac{1}{(1-t)^{\propto+1}}^{\frac{-t^x}{1-t}}$$

(b) Define generalized Laguerre polynomial.

Q.4 (a) Define $T_n(x)$ and show that

$$\sum_{n-0}^{\infty} T_n(x) = \frac{1-xt}{1-2xt+t^2}$$

(b) Prove Orthogonality of Jacobi polynomial.

OR

(a) Show that $\frac{1}{\sqrt{1-x^2}} U_n(x)$ satisfies the differential equation.

$$(1-x^2)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + (n^2-1)u = 0$$

(b) Show that

$$P_n^{(\propto,\beta)}(\text{-z}) = (-1)^n P_n^{(\beta,\propto)}(z)$$

Q.5 (a) Show that

$$\int_{-1}^{1} T_n(x)T_m(x) = \frac{dx}{\sqrt{1-x2}} \begin{cases} o & n \neq m \\ n & n = m = o \\ \frac{n}{2} & n = m \neq o \end{cases}$$

(b) State and prove Bateman's generating relation of Jacobi polynomial.

OR

(a) Show that

$$\int_0^{\infty} e^{-x}x^k l_n(x) L_m{}^k(x)dx = \frac{(n+k)!\, smn}{n!}$$

(b) Show that

(i) $He_{n+1}(x) = xHe(x) - 2nHn - 1\,(x)$

(ii) $He_{n \quad +1}(x) = 2xHn(x) - 2nHn - 1\ (x)$

Enrollment No……………..

Master of Science (Mathematics)
Fourth Semester Main Examination, June-2021
Theory of linear Operator-II [MSM403T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Define projection operator. Also prove that a bounded liner operator P on a Hilbert space H is a projection if P is self adjoint and idempotent.

(b) Show that the operator T, defined by $T(x) = f(x)z$ is compact, where z be any fixed element of x and $f \in x$

OR

(a) Let T_n be a sequence of compact linear operator, from a normed space x into a Banach space y. If T_n is strongly operator convergent, then prove that T is compact.

(b) Define : - (i) Relatively compact
(ii) Totally boundeness
(iii) Spectrum

Q.2 (a) Let T be a bounded, self adjoint linear operators on a complex Hilbert space H and $\sigma(T)$ is the spectrum of T then prove that

$$\sigma(T)C[m, M] \; where \; m = \inf_{\|x\| = 1} < T_x, x > and \; M = \sup_{\|x\| = 1} < T_x, x >$$

(b) Let T is compact liner operator defined on the normal space x then prove that N (Tλ) is finite dimensional with λ≠o.

OR

(a) Let T: y be a compact liner operator in bounded then prove that T is continuous Discuss the case where T is not compact.

(b) State and prove fredholum alternative theorem.

Q.3 (a) Let T:D (T) => H be density defined linear operator in H and suppose that T is injective and its range R (T) is dense in H then prove that T* is injective. And $(T^*)^{-1} = (T^{-1})^*$

(b) If two bounded self adjoint linear operators S and T on the Hilbert space H are positive and commute then prove that ST is positive.

OR

(a) Let T: x => x be a compact linear on normed space x and ≠o then show that for the smallest ineger depending with such that from n=r on the null space N(T_λ^n) are all equal and if r>o the inclusion N(T_λ) ⊂ N(T_λ^1) ⊂ ………. N(T_λ^r) are all proper.

(b) Prove that every positive bounded self adjoint linear operator T on a complex Hilbert space H has a positive square root and is unique.

Q.4 (a) Let T be compact linear operator defined on normal space x and for non-zero value λ≠o. prove that range of Tλ is closed.

(c) Prove that production and summation of projection is also a projection.

OR

(a) If T is linear operator defined on a complex Hilbert space H and symmetric condition then prove that T is bounded.

(b) Let B be a subset of a metric space x If B is totally bounded then prove that B is finite ∈- net and B is seperable.

Q.5 (a) Define closable operator and minimal extension.Also Let T:D (T) → H be densely defined linear operator in H, if T is self adjoint then prove that T is symmetrical but converse is not ture.

(b) Define residual spectrum. Also state and prove residual spectrum theorem.

OR

(a) Let $S: D(S) \to H$ and T:D(T) → H be linear operators. Which are densely defined in a complex Hilbert space H then prove that If $D(T^*)$ is dense in H then $T \subset T^{**}$.

(b) Let T:x → Y is a continuous mapping of a metric space x into a metric space Y then prove that the image of a relatively compact set A⊂ × is relatively compact.

Enrollment No……………..

Master of Science (Mathematics)
Fourth Semester Main Examination, June-2021
Advanced Numerical Analysis-II [MSM404T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Solve the initial problem
u = t^2- u^2, u(0) =1 estimate u (0.3) using third order Adams- Bash forth method with h=0.1 obtain the starting Values the third order taylor series method.

(b) Find the Jacobian Matrix for the system of equations.

$f_1(x,y) = x^2 + y^2 - x = 0,\quad f_2(x,y) = x^2 - y^2 - y = 0$ at the point (1,1) with $h = R = 1$.

OR

(a) The second order difference method is used to solve the differential equation $\mu'' + w^2\mu = 0$. Show that the solution of the difference equation.

(b) Explain Richardson's extrapolation.

Q.2 (a) Let P (∈) = (∈-1) (∈-h) where h is real and -1<h<1 find σ(∈) to determine implicit methods. Write the methods explicitly for h=0 and h=1.

(c) Explain stability of PHp CMc mehod.

OR

(a) Test the stability of second order nystrom method to the test equation $\mu' = \lambda\mu$.

(b) Prove that the order P of an A-stable linear multistep method cannot exceed 2 and the method must be implicit.

Q.3 (a) Derive the general solution of linear second order differential equation by shooting method for boundary condition of first kind.(case 1 and case 2)

(b) Using the shooting method solve the first boundary value problem

$$\mu'' = \mu + 1,\quad 0 < x < 1$$
$$\mu(0) = \mu(1) + \mu'(1) = e - 1$$

OR

(a) Explain linear second order differential equation by shooting method for boundary condition of the second kind and third kind.

(b) Use the shooting method to solve the mixed boundary value problem

$$\mu'' = \mu - 4xe^{x}, \quad 0 < x < 1$$
$$\mu(0) = \mu'(0) = -1 \qquad \mu(1) + \mu'(1) = -e$$

Use Taylor series method to solve the initial value problems Assume h=0.5

Q.4 (a) Use a second order method for the solution of the boundary value problem.

$$\mu'' = x\mu + 1, \quad 0 < x < 1$$
$$\mu(0) = \mu'(0) = 1, \quad \mu(1) = 1 \text{ with step length h=0.25}$$

(d) Solve the boundary value problem

$$\mu' = \mu' + 1$$
$$\mu(0) = 1, \qquad \mu(1) = 2(e-1)$$
$$with\ h = \frac{1}{3} \quad using\ second\ order\ method.$$

OR

(a) Solve the boundary value problem. $\mu' = 4\mu + 3x\ ,\ 0 < x < 1$

$$\mu(0) = , \qquad \mu(1) = 1$$
$$with\ h = 0.25\ using\ second\ order\ method.$$

(b) Solve the boundary value problem

$$\mu'' = \mu + x$$
$$\mu(0) = 0,\ \mu(1) = 1 \quad with \quad h = \frac{1}{4}\ by\ second\ order\ method$$

Q.5 (a) Derive the matrix form of the Variational equation of linear boundary value problem. $\frac{-d}{dx}[p(x)\mu'(x)] + q(x)\mu(x) = r(x)$

Subject to the boundary condition of first kind $\mu(a) = y_1,\ \mu(b) = y_2$ by finite element method.

(b) Explain Ritz method for boundary value problem.

OR

(a) Consider the boundary value problem

$$\mu'' + 2\mu = x, \quad 0 < x < 1$$
$$\mu(0) = 0\ , \qquad \mu(1) = 1$$

Determine the coefficient of the approximate solution function $w(x) = x(1-x)(a_1 + a_2 x)$ by the Ritz method.

(b) Solve the boundary value problem.

$$\mu'' + \mu = x, \quad 0 < x < 1$$
$$\mu(0) = 4\ , \qquad \mu'(1) = 1$$

Using the Ritz finite element method with linear piecewise polynomial for two elements of equal lengths

Enrollment No………………

Master of Science (Mathematics)
Fourth Semester Main Examination, June-2021
Integral Transform-II [MSM407T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Find the laplace transform of the triangular wave function.

$$f(t) = \begin{cases} t & 0 < t < a \\ 2a - t\ , & \frac{0}{a} < t < 2a \end{cases}$$

(b) Evaluate $L\left\{ 1 - \frac{cos2t}{t} \right\}$

OR

(a) Prove that $L\left\{\int \frac{sint}{t} \right\}$ = tan−1 $\frac{1}{P}$ and Fine L$\left\{ \frac{sinat}{t} \right\}$ Does laplace transform of $\frac{\cos at}{t}$

(b) Find the value of $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$.

Q.2 (a) If the switch is connected at t=0 and discounnected at t= a. find the current I in ferm of t

Find $L\{G(t)\}$ where $G(t) = \begin{cases} e & t > a \\ 0\ ; & < ta \end{cases}$

(b) Find the La Place transform of

$$\frac{Cos\ at - Cos\ bt}{t}$$

OR

(a) Evaluate $L\left\{\int_0^t \frac{sint}{t}\ dt\right\}$

(b) Find laplace transform of t^2 Cosat.

Q.3 (a) State and prove Inversion formula for fourier complex transform
(b) Find the cosine transform of the function f (x) ; if

$$F(x) = \begin{cases} Cosx\ 0 & < x < a \\ 0 & x > a \end{cases}$$

OR

(a) Define Fourier transform and explain the shifting property of fourier transform..

(b) Find the Fourier complex transform of f (x) if

$$F(x) = \begin{cases} e & 0 < x < b \\ 0 & x < a, x > b \end{cases}$$

Q.4 (a) Show that

$$\int_0^\infty \frac{sint - t\ c\theta st}{t^3})2\ dt = \frac{\pi}{15}$$

(b) State and prove convolution theorem.

OR

(a) Using the convolution theorem find

$$L^{-1}\left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}\right]$$

(b) Evaluate show that the fourier transform of

$F(x) = \begin{cases} a^2 - x^2 & 1x1 < a \\ 0 & 1x1 > a \end{cases}$ is $\frac{2\sqrt{2}}{n}$ $(\frac{\sin as - as\ c\theta s\ as}{s^3})$ using parseval's indentify

Q.5 (a) Find the fourier cosine transform of $F(x)$ = $5e^{2x}$+ $2e^{5x}$

(c) Find Laplace transform of t^2e^t sin4 t.

OR

(a) Find $f(x)$ if its finite sine transform is given by

$\frac{25(-1)^{p-1}}{p^3}$, where P is positive integer and 0 <x <n.

(b) Find the fourier cosine transform of.

$$f(x) = \frac{e^{-ax} - e^{-bx}}{x}$$