Enrollment No……………..

Master of Science (Mathematics)

Second Semester Examination, June-2021

Advanced Abstract Algebra-II [MSM201T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Each question has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Write down three Application of fundamental structure theorem.
(b) Define Abelian groups with suitable example.

OR

(a) State and Prove Weddeburn- Artin theorem.
(b) Define Artinian ring with example.

Q.2 (a) Prove that A Subring of Noetherian ring need not to be Noretherian.
(b) State and Prove Hilbert Basis theorem.

OR

(a) What do you mean by free modules, Explain with example?
(b) State and Prove Primary decomposition theorem.

Q.3 (a) Let M be a finitely generated free module over a commutative ring R. then show that all bases of M are finite.
(b) Define simple (or irreducible) with example.

OR

(a) Define Isomorphism with example.
(b) (a) Let M be an R-module. Show that the set $\{X \in R/xm = 0\}$ is an Ideal of R.

Q.4 (a) Define Isomorphism with example.
(b) Define R- homomorphism with. Example

OR

(a) Let R be a ring with unity and M be an R-Module. Then show that the following conditions are equivalent.
i) M is simple.
ii) M $\neq$ (0) and M is generated by any x $\neq$ 0 belonging to M.

(b) Let M be a finitely generated free module over a commutative ring R. then show that all bases of M have the same number of elements.

Q.5 (a) Let M be an R-Module and N_i ($1 \leq i \leq t$) be R-Sub modules of M. Show that the smallest sub module of M containing $U_{i=1}^{t}$ N_i is $\{ x_1 + \_\_\_\_\_ + x_t / x_i \in N_i , 1 \leq i \leq t\}$

(b) Define direct sum of R-Sub Modules of an R-Module M. Show that the following conditions are equivalent.

i) $\sum_{i=1}^{t} N_i$ is direct.

ii) if $x_1+x_2+\_\_\_\_\_ x_t = 0$ Where $x_i \in N_i$ then $x_i = 0$ for all i.

OR

a) Define R-module homomorphism. Show that Hom (M, N) the set of all R-Module homomorphism of M to N is an R-Module if R is a commutative ring.

b) Prove that when R is considered as a module over itself submodules are ideals.

Enrollment No……………..

Master of Science (Mathematics)
Second Semester Examination, June-2021
Lebesgue Measure and Integration [MSM202T]

Time: 3:00 Hrs | **Max Marks 85**

Note: Attempt all questions. Each question has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Explain Uniform Convergences.
(b) Let F and g be non- negative measurable function then show that

$$\int (f+g) = \int (f) + \int (g)$$

Q.2 (a) Explain Lebesgus measurability.
(b) Show that the class of Lebesgue measurable set is a σ Algebra.

Q.3 (a) State and Prove Lebesgue differentiation theorem.
(b) State and Prove Jensen's Inequality.

Q.4 (a) Explain and prove Lebesgue's monotone convergence theorem.
(b) Explain function of Bounded Variations.

Q.5 (a) What do you mean by Non- Measurable sets?
(b) State and Prove Holder Inequalities.

Q.6 (a) Explain regularity.
(b) State and Prove MiniKowski Inequality.

Q.7 (a) Show that the outer measure of an Interval is its length
(b) What do you mean by convex functional? Explain.

Q.8 (a) State and Prove Fatou's Lemm.
(b) Explain dual of LP.

Enrollment No……………..

Master of Science (Mathematics)
Second Semester Main Examination, June-2021
Topology-II [MSM203T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Each question has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Show that a finite subset of T1 space X has no accumulation points.

(b) Prove that is A and B are subsets of topological space (x, τ), then $(A \cup B)' = A' \cup B'$

Or

(a) Let $X = \{a, b, c, d, e\}$ Test whether $\tau_1 = \{x \emptyset \{a, b, c\}, \{a, b, d\}, \{a, b, d\}, \{a, b, c, d\}\}$ is topology on x.

(b) Prove that the components of a totals disconnected space X are the singleton subset o

Q.2 (a) Define a Compact topological space show that continuous image of compact sets are compact.

(b) Show that every closed subset of a compact space is also compact.

Or

(a) Show that every second countable space is also first countable.

(b) Let F1 and F2 be disprint closed subset space X. Then show that there exists a continuous function $f: X \rightarrow [0,1]$ such that $f[f_1] = 0$ and $f[f_2] = 1$ compact.

Q.3 (a) State and Prove Weierstrass approximative theorem.

(b) Show that a totally disconnected space is Hausdorff.

Or

(a) Show that if [Aj] B a class of connected subsets of X such that no two members of if one separated then UAj is connected.

(b) Show that a complete regular space is also regular.

Q.4 (a) Prove that every second countable normal T1 space is metrizable.

(b) Prove that the Euclidian space R^m is connected.

Or

(a) Prove that an infinite set with the co-finite topology is a T1-space.

(b) Define T2-space show that T1-space does not implies T2-space

Q.5 (a) Prove that if the components of a compact space are open then there are only a finite number of then.

(b) If A is a compact subsets of Hausdorff space X and P $\notin A$ then there is an open set G. such that $P \in G\ C\ A$.

Or

(a) If $f: x \rightarrow y$ be a continues function and then $A\ C\ x$ show that $f/A: A \rightarrow y$ is continues. Is the converse True? Justify.

(b) Prove that every closed subset of a locally compact space is locally compact.

Enrollment No……………..

Master of Science (Mathematics)
Second Semester Main Examination, June-2021
Complex Analysis-II [MSM204T]

Time: 3:00 Hrs **Max Marks 85**

Note: (i) Attempt all questions.
(ii) All Questions carry equal marks.

Q.1 (a) State and Prove Weierstrass factorization theorem.
(b) State and prove mean value theorem.

Q.2 (a) State and Prove Runge's theorem.
(b) State and Prove Mittay –Leffler's theorem.

Q.3 (a) Explain space of Analytic function with example.
(b) State and Prove Hurwitz's theorem.

Q.4 (a) Show that $U = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and find its harmonic conjugate.
(b) Explain briefly Schwartz Reflection Principle.

Q.5 (a) What do you mean by Analytic continuation along a curve? Explain.
(b) State and Prove Montel's theorem.

Q.6 (a) Explain Range of Analytic function.
(b) What do you mean by Canonical Products?

Q.7 (a) State and Prove Schottky's theorem.
(b) State and Prove Bloch's theorem.

Q.8 (a) State and prove Riemann mapping theorem.
(b) State and prove the Hadmard's three circle theorem.

Enrollment No……………..

Master of Science (Mathematics)
Second Semester Main Examination, June-2021
Differential Equation-II [MSM205T]

Time: 3:00 Hrs **Max Marks 85**

Note: Attempt all questions. Each question has two parts. Part A is 10 marks and part B is 7 marks.

Q.1 (a) Write three principal of properties solution.
(b) Define Banach fixed point theorem with example of fixed point.

Q.2 (a) What are Non-Oscillatory equation, Explain with the help of example.
(b) State and Prove Non-Oscillator theorem.

Q.3 (a) Define Stationary point, what do you mean by Index of a Stationary point
(b) Explain parameters preliminaries.

Q.4 (a) What are linear and non- linear equations. Write two examples of each.
(b) Write uses of Implicit function.

Q.5 (a) Explain Autonomous system with example.
(b) State and Prove Poincare- Bendixson theorem

Q.6 (a) Write the basic facts of linear Second order equation.
(b) Explain limit and Integral properties of principal solution.

Q.7 (a) State and Prove Sturm theorem.
(b) Write down the properties of Sturm- Liouville boundary value problem.

Q.8 (a) Explain Number of Zero's.
(b) Solve the equation
$y'' = x + y$ with the boundary condition $y(0) = y(1) = 0$