Enrollment No……………..

**Master of Science (Mathematics)**
**Third Semester Main Examination, Dec-2020**
**Advanced Numerical Analysis-I (MSM-304T)**

**Time: 3:00 Hrs** **Max Marks 85**

**Note : Attempt all questions. Question no. 1 to Q. no. 5 has two parts. Part A is 10 marks and part B is 7 marks.**

Q.1 (a) Using the method of linear interpolation find y at x = 0.5 and x = 0.75
Given the following table-

| | | | | |
|---|---|---|---|---|
| x | 0 | 1 | 2 | 5 |
| y | 2 | 3 | 12 | 147 |

(b) Explain Hermite interpolation.

OR

(a) Fit the linear spline to the following data-

| | | | | |
|---|---|---|---|---|
| x | 1 | 3 | 6 | 8 |
| y | 4 | 5.5 | 7 | 9.5 |

Estimate the value of x = 2, 4, 7

(b) Explain quadratic spline interpolation.

Q.2 (a) Find the value of y at x=0
Given some set of values (-2, 5) (1, 7) (3, 11) (7, 34) ? by Lagranges interpolation formula.
(b) Write three application of Newtonis Bivariat interpolation polynomial.

OR

(a) Fit a straight line for the following data-

| | | | | | |
|---|---|---|---|---|---|
| x | 2 | 4 | 6 | 8 | 10 |
| y | 3 | 7 | 9 | 11 | 17 |

(b) Explain discreet and continous data.

Q.3 (a) Prove tha $n^{tn}$ Legendre polynomial $P_n$ is given by –

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2-1)^n]$$

(b) Write the formula for chebyshev polynomial.

OR

(a) Explain Gram – Schmidt orthogonalization.
(b) Prove shifted chebyshev first kind.

Q.4 (a) Prove that uniform approximation by polynomial when sets are not compact.
(b) Explain Rational approximation.

OR

(a) What do you mean by choice of methods.
(b) Evaluate $\Delta^n\left(e^{ax+b}\right)$

Q.5 (a) Find the first & second derivative of f(x) at x = 1.5

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| y | 3.375 | 7 | 13.625 | 24 | 38.875 | 59 |

(b) Explain Non – uniform nodal points.

OR

(a) Find the first derivatie at x = 4.5 for the following data –

| | | | | | |
|---|---|---|---|---|---|
| x | 2 | 4 | 6 | 8 | 17 |
| y | 3 | 8 | 8 | 11 | 13 |

(b) Show that $E \equiv 1 + \Delta$ or $\Delta \equiv E-1$
Also write their usual meaning.

Enrollment No……………..

**Master of Science (Mathematics)**
**Third Semester Main Examination, Dec-2020**
**Advanced Special function-I (MSM302T)**

**Time: 3:00 Hrs** **Max Marks 85**

**Note: Attempt all questions. Each question has two parts. Part A is 7 marks and part B is 10 marks.**

Q.1 (a) Prove that $\Gamma 1 = 1$ and $\Gamma(n+1) = n\Gamma n$ and $\Gamma\frac{1}{2} = \sqrt{\pi}$
(b) State and Prove Legendre's duplication formula.

OR

(a) Prove that $\beta(m,n)=\frac{\sqrt{m}\sqrt{n}}{\sqrt{m+n}}$
(b) State and Prove Gauss Multiplication theorem.

Q.2 (a) Define hyper geometric function.
(b) Prove that ${}_2f_1\left(\frac{a,b\,;\,z}{c}\right)=(1-z)^{-a}\ {}_2f_1\left(\frac{a,c-b}{c};\frac{z}{z-1}\right)$

OR

(a) Find the inverse of $x=\begin{bmatrix} 2 & 4 \\ 5 & -1 \end{bmatrix}$
Hence solve the simultaneous equations.
2x+4y=1
5x-y=8
(b) Define simple transformation and quadratic transformation. Also show that f(x) is a quasi concave (quasi convex) on $T_{++}$ and $T_{--}$ ($T_{+-}$ and $T_{-+}$)

Q.3 (a) Define contiguous function.
(b) State and prove Dixon's theorem.

OR

(a) State and prove Whipple's theorem.

(b) State and prove Ramanujan's theorem.

Q.4 (a) Prove that

(i) $J'n(x) = J_{n-1}(x) - J_{n+1}(x)$

(ii) $xJ_n{}'(x) = xJ_{n-1}(x) - nJ_n(x)$

(b) Define Bessel's differential equation.

OR

(a) Show that

$J_{3/2}(x) = \sqrt{\frac{2}{\pi}}\left(\frac{sinx}{x} - cosx\right)$

(b) Show that Bessel functions Jn(x) are linked together by the relation

$e^{\left(\frac{x}{2}\right)\left(.t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} \mathrm{Jn}(x)t^n$

Q.5 (a) Express $f(x) = x^4 + 2x^3 + 2x^2 - x - 2$ in terms of Legendre's polynomial.

(b) Obtain Rodrigue's formula for Legendra polynomial.

OR

(a) Prove that $\int_{-1}^{1} p_n\ (x)dx = 0\ \ if\ \ n \geq 1$

(b) If $p_n$ is a Legendre's polynomial of degree n then to prove the relation

$(2n+1)xp_n = (n+1)p_{n+1} + np_{n-1}$

Enrollment No……………..

**Master of Science (Mathematics)**
**Third Semester Main Examination, Dec-2020**
**Functional Analysis-I (MSM-301T)**

**Time: 3:00 Hrs** **Max Marks 85**

**Note : Attempt all questions. Question no. 1 to Q. no.4 has two parts. Part A is 10 marks and part B is 7 marks. Q.5 is of 17 marks.**

Q.1 (a) Let n be a Normed linear space and let x, y∈N then

$\| x \| - \| y \| \leq \| x - y \|$

(b) Define Normed linear spaces. What do you mean by complex normed linear space.

OR

(c) Show that the real linear space R and the complex linear space C are Banach spaces under the norm $\| x \| = |x|, x \in c\ (R)$

(d) Define Banach space with example.

Q.2 (a) Prove that $R^n$ is an nls (normed linear space) with the following norms.

$\| x \|_1 = \sum_{i=1}^{n} |x_i|$

(b) State Riesz lemma.

OR

(a) What do you mean by finite dimensional normed spaces and subspaces.

(b) Explain linear operator.

Q.3 (a) Prove that if a linear operator is continuous then it is bounded.

(b) Define bounded linear operator with example.

OR

(a) Define linear functional with two example.

(b) Define functional with example.

Q.4 (a) What is linear operator? What is difference between a linear operator and linear transformation?

(b) Explain dual spaces with example.

OR

(c) Prove that every finite dimension normed linear space is a Banach space.
(d) Explain Banach space.
(e) Q.5 Write short note on any three-
(f) i) Complex Vector Space
(g) ii) Any two application of bounded linear functional on c [a, b]
(h) iii) Quotient Space
(i) iv) Compactness
(j) v) Normed Space

Enrollment No……………

**Master of Science (Mathematics)**
**Third Semester Main Examination, Dec-2020**
**Integration Theory (MSM309T)**

**Time: 3:00 Hrs** **Max Marks 85**

**Note : 1.Attempt all questions.**
**2. Question No 1 to Question No 4 has 2 parts (Parts A is 10 Marks & Part B 7 Marks. Question five is of 17 Marks.**

Q.1 (a) Prove that every finite measure is semi-finite.

(b) Define measure with example.

OR

(a) If $F_1$ and $F_2$ are measure function then the function $F^*$ and $F_*$ are measurable.

(b) Define sigma finite set with example.

Q.2 (a) Show that the function ψ defined on R by.

$$\Psi(x) = \begin{cases} x+5 & if\ x < -1 \\ 2 & if -1 \le x < 0 \\ x^2 & if\ x \ge 0 \end{cases}$$

(b) What do you mean by measurable function.

OR

(a) Show that constant function with a measurable domain in measurable.

(b) Define step function with example.

Q.3 (a) If F is measurable; g is Integrable and $|f| \le |g|$ then f is integrable.

(b) Define characteristic function; what do you mean by indicator.

OR

(a) A mean fundamental sequence $\{f_n\}$ of integrable function is fundamental is measure.

(b) Define mean fundamental.

Q.4 (a) What are signed measure? Explain with example.
(b) State Hahn decomposition theorem.

OR

(a) What do you mean by mutually singular measure. Explain.
(b) State Jordan decomposition theorem.

Q.5 State and prove Radon Nykodym theorem.

OR

State and prove Lebesgue decomposition theorem.

Enrollment No……………..

**Master of Science (Mathematics)**
**Third Semester Main Examination, Dec-2020**
**Theory of Linear Operators-I (MSM-303T)**

**Time: 3:00 Hrs** **Max Marks 85**

**Note : Attempt all questions. Question no. 1 to Q. no.5 has two parts. Part A is 10 marks and part B is 7 marks.**

Q.1 (a) Prove that Normal linear space is a linear matic space with respect natural metric defined by
d $(x, y) = \| y - x \|$
(b) Define closed operators with example.

OR

(e) Let X be a normal space and T:X →X be a compact lenear operator, If dimX = ∞ then show that 0 $\epsilon$ $\sigma$(T)
(f) What do you mean by continuous and residual spectra.

Q.2 (a) Write spectral properties of bounded linear operator.
(b) What do you mean by spectrum; Explain with one-example.

OR

(a) State and prove spectral mapping theorem for polynomial.
(b) Write only three properties of resolvent.

Q.3 (a) Let X be complex Banach division algebra, then prove that X is isomorphic to c.
(b) Write three properties of Banach Algebra.

OR

(a) If X be a Banach Algebra and let x $\epsilon$ X the r (x) = $\lim_{x\to\infty} \|x^n\|^{1/n}$
(b) If X is Banach Algebra and x $\epsilon$ X then $\sigma$ (x) ≠ ∅

Q.4 (a) Prove that bounded linear operator o finite rank is compact.
(b) Define compact operator with example.

OR

(k) If T : $L_2$ [a, b] → $L_2$ [a, b] be an operator with separable kernel then T is compact.
(l) State compact integral operator.

Q.5 (a) Prove that $|\langle x,y\rangle| \leq \|x\|.\|y\|$ where x, y belongs to Hilbert space H.

(b) Write any three spectral properties of compact linear operator.

OR

(a) Prove that every linear space X has Hamel Base.

(b) Define normal linear space with example.